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THE OUTLINES OF THE POLYHEDRIC WORLD

SUMMARY

Although we do not always realise this, the shape of most of the visible world around us is to a great extent governed by the geometry of polyhedra. A polyhedron is a shape that is covered by many (poly) flat faces (hedra). Even curved surfaces can often be considered as a three-dimensional tiling of infinitesimally small plane faces. If we use in this context the term 'polyhedron' we are generally referring to the so-called Platonic and Archimedean solids, which are convex bodies that are covered by a closed pattern of regular polygons. They have a form that is so perfect, that they exert a great attraction to both artists and technicians. Also in architecture they have been applied in many ways and they form the geometric basis of most buildings and structures This paper deals specifically with the architectural use of these forms and with their influence on our man-made environment. They can either define the overall shape of the building structure or its internal configuration

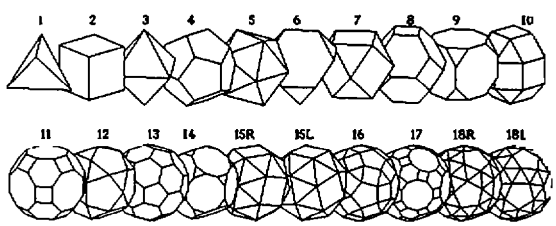

Fig. 1. Review of the regular (Platonic) and the semi-regular (Archimedean) polyhedra.

The numbers P with an index, that are given here, will further be used as a reference.

THE PLATONIC AND THE ARCHIMEDEAN SOLIDS

The polyhedra that are considered here, comprise the 5 regular solids that are ascribed to Plato and the 13 semi-regular solids that are said to have been discovered by Archimedes. Two of them have 'enantiomorphic' or left- and right-handed versions: P15 and P18, the snub cube and the snub dodecahedron.

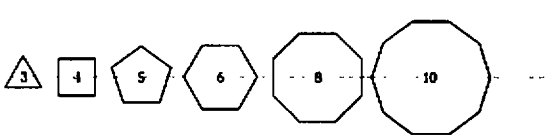

Fig. 2. The 6 different regular polygons that constitute the polyhedra of Fig. 1

These solids all are composed of 1, 2 or 3 kinds of regular polygons. All vertices lie on a circumscribed sphere and all vertex situations are identical, which means that the polygons always meet in these vertices in the same order of sequence. P7 and P12 are also called 'quasi-regular', because of the great regularity in their vertex situations: always 4 polygons in the combinations 3-4-3-4 or 3-5-3-5.

THE SNUB POLYHEDRA, A SPECIAL CASE

The two solids with the left and right variants P15 and P18 are derived by double truncation of the cube or the dodecahedron. They consist of either a square or a pentagon and a number of triangles, which are arranged in the form of caps in numbers corresponding with the vertices of the circumscribed solids. The icosahedron can by analogy be considered as a snub tetrahedron.

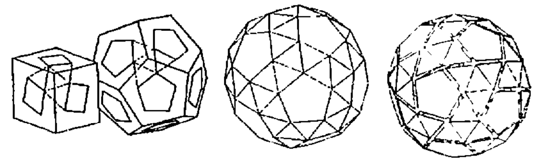

Fig. 3. The construction of a snub polyhedron.

THE RECIPROCAL OR DUAL SOLIDS

Each of these solids has its counterpart. It is found by choosing a point above each polygon and connecting it to those of all of its neighbours, so that the connecting lines bisect the polygon edges perpendicularly and that they also are perpendicular to the line from the midpoint of the edge to the centre of the polyhedron. The faces that thus are formed in such a reciprocal solid are flat and identical. This phenomenon is called duality. Two of these are well-known: the rhombic dodecahedron (R7) and the rhombic icosahedron (R12), that are derived from the quasi-regular solids. The reciprocals have the name R with an index number, corresponding to their related polyhedron.

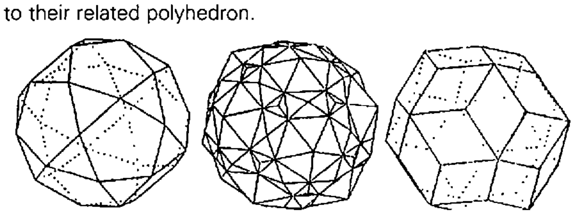

Fig. 4. The principle of duality, demonstrated here for P12

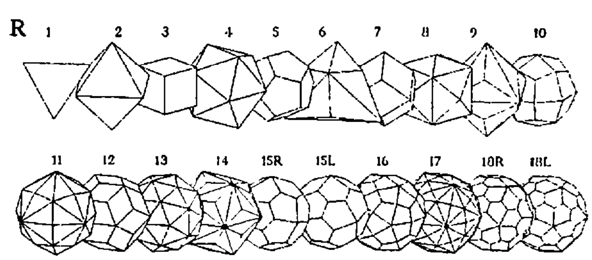

Fig. 5. Review of the reciprocal solids R1 to R18

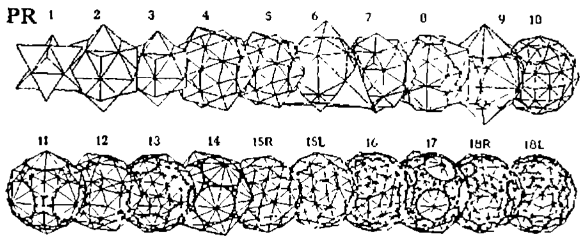

Fig. 6. Review of the compounds formed by the polyhedra and their reciprocals.

PACKINGS AND SPACE FRAMES

Polyhedra lend themselves to be put together in tight packings. That makes them suitable as the basic configuration for space frames, because of their great uniformity: identical members meeting under specific angles.

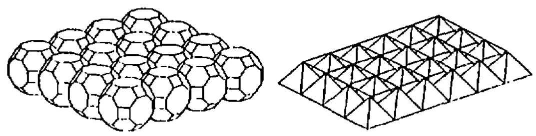

Fig. 7. Packing of P11's and another of half-octahedra (P3) and tetrahedra (P1).

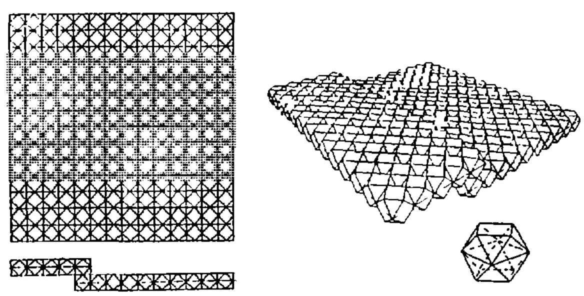

Fig. 8. A packing of the quasi-regular P7, suggested for a hangar roof.

THE STELLATED OR KEPLER-POINSOT POLYHEDRA

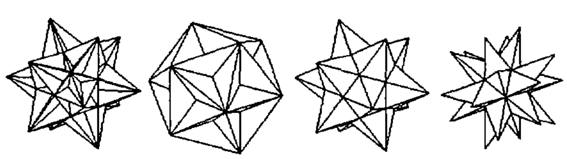

Fig. 9. The four regular star-polyhedra, indicated here as C1 to C4.

Another group of regular figures that specially must be mentioned is that of the star-shaped polyhedra, in this context indicated as C1 to C4. They have faces that are formed by intersecting pentagonal star-polygons or pentagrams. C1 can also be considered as an intersection of 20 triangles and C2 of 12 pentagons. Hence they are called the great icosahedron and the great dodecahedron. C1 and C2 were discovered by Poinsot and the others by Kepler.

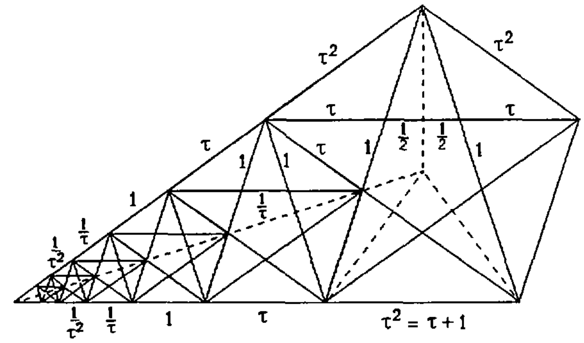

Fig. 10. A sequence of pentagrams, having a relationship of ((1 + (5)/2

PRISMS AND ANTIPRISMS

The regular prismatic and antiprismatic solids have similar characteristics as the uniform polyhedra, but they form in fact endless rows They are defined by two parallel n-gons The prisms have mantles that are formed by a row of squares

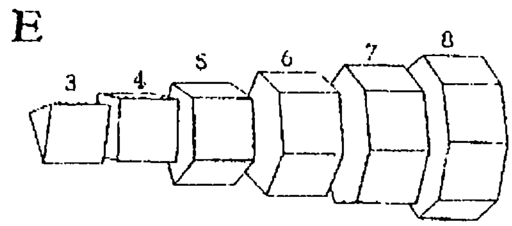

Fig. 11. Rows of prisms with polygons or polygrams as the defining factor.

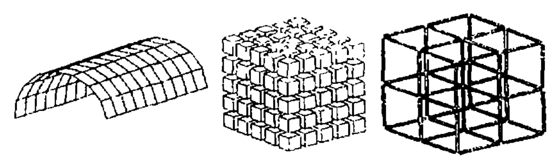

Fig. 12. A few examples of prismatic shapes.

The antiprisms have triangular circumferential faces, and their n-gons are rotated with respect to each other. Both prisms and antiprisms have also star-shaped versions.

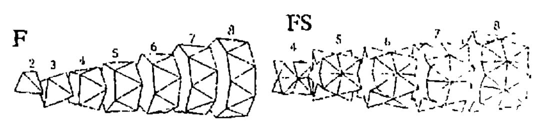

Fig. 13. Row of polygonal and star-shaped antiprisms.

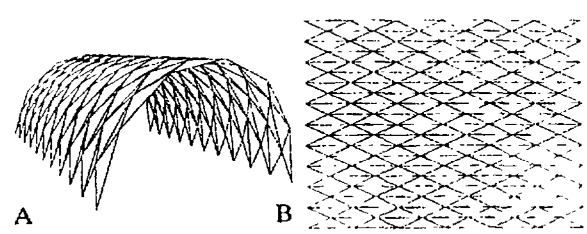

Fig. 14. A typical antiprismatic structural form (B is A, in folded flat position).

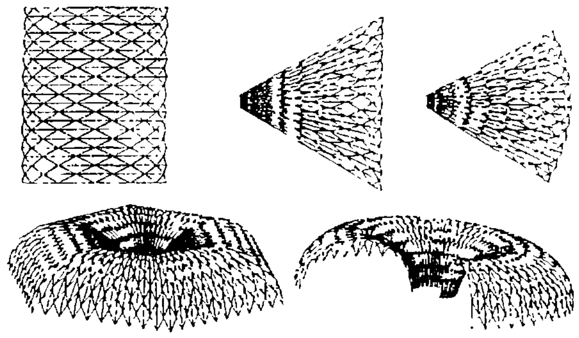

Fig.15. Exercises with form A as starting the point.

Fig. 16. Same form A as in Fig. 14 with added-on quarter spheres

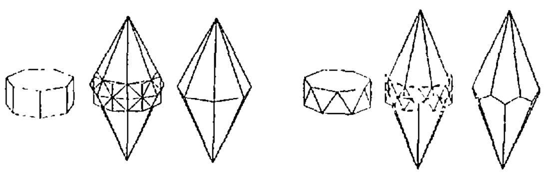

Fig. 17. Both groups have reciprocal counterparts also.

ADDITIONS TO POLYHEDRA

Upon the regular faces of the polyhedra other figures can be put, as long as they have the same basis. In this way polyhedra can f.i. – so to say – be 'pyramidized'. This means that shallow pyramids are put on top of the polyhedral faces, having their apexes on the circumscribed sphere of the whole figure. This can be considered as the first frequency subdivision of spheres.

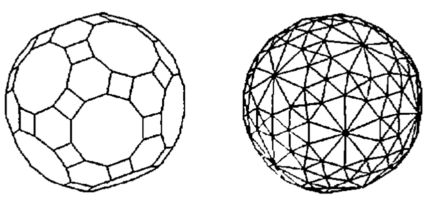

Fig. 18. 'Pyramidized" version of P17.

In 1582 S Stevin introduced the notion of 'augmentatation' by adding pyramids, consisting of triangles and having a triangle, a square or a pentagon as its basis, to the 5 regular polyhedra. Recently, in 1990 D.G. Emmerich extended this idea to the semi-regular polyhedra.

He suggested to use also pyramids of 6–, 8– or 10–sided plan and that themselves are composed of regular polygons. There are 7 of such pyramids, that are suitable for this purpose and that in fact are parts of other polyhedra. Emmerich found out that they can be combined to form 102 different combinations He calls these: composite polyhedra.

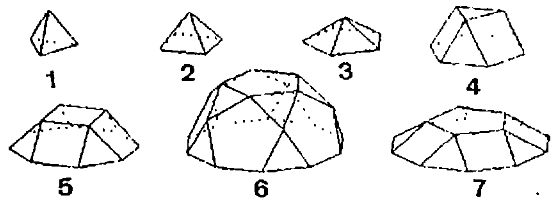

Fig. 19. 7 regular pyramids, suitable for augmentation

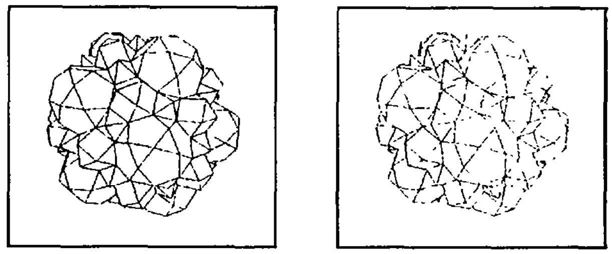

Fig. 20. Stereoscopic pair of augmented P17

GEODESIC DOME SUBDIVISIONS

Any of the polyhedra can be used as the starting point for a more refined subdivision pattern of spherical forms, that are so often used in architecture.

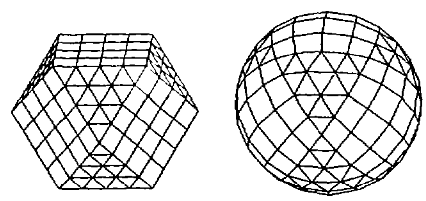

Fig. 21. Example of subdivision pattern on the basis of P7

Subdivision patterns are written on the faces of these figures and the co–ordinates of the intersection points can be converted from cartesian to polar co–ordinates. If all distances are then taken equal to the radius of the circumscribed circle, the – originally polyhedric – form is turned into a sphere.

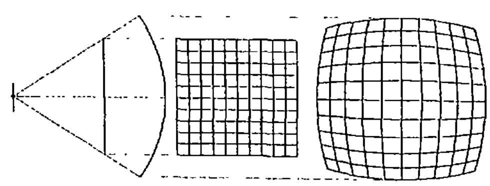

Fig. 22. Conversion of polyhedron co–ordinates to spherical co–ordinates.

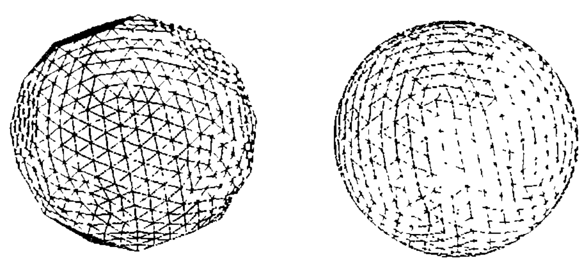

Fig. 23. Sphere subdivision on the basis of the snub dodecahedron P18

Truncations lead often to curved lower boundaries For building purposes it is necessary to adapt the vertical co-ordinates of the under-most points to those of the cutting plane

Fig. 24. Adaptation of lower boundary of icosahedral cap to horizontal plane

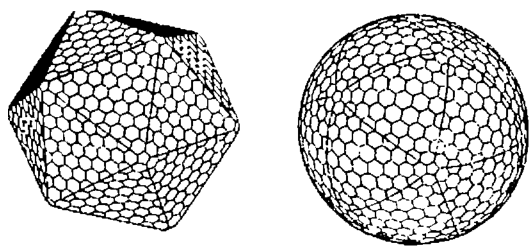

Fig. 25. Any pattern is thus produceable, like this hexagonal tiling on an icosahedron

In this way also the dual or reciprocal figures of the polyhedra and the prisms and antiprisms can be subdivided and converted to spheres.

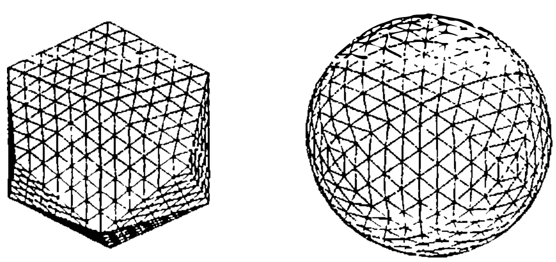

Fig. 26. Subdivision of 'reduced' dual.

The vertices of this rhombic dodecahedron (R7) are first brought on one circumscribed sphere, before the subdivision and successive conversion take place.

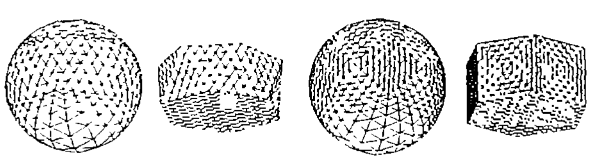

Fig. 27. The hexagonal prism and antiprism, used as the basis for a spherical subdivision.

ACKNOWLEDGEMENT

The accompanying figures have been made with the computer programme CORELLI, that is being developed by the author at the Delft University of Technology in close co-operation with G van der Ende who did the programming