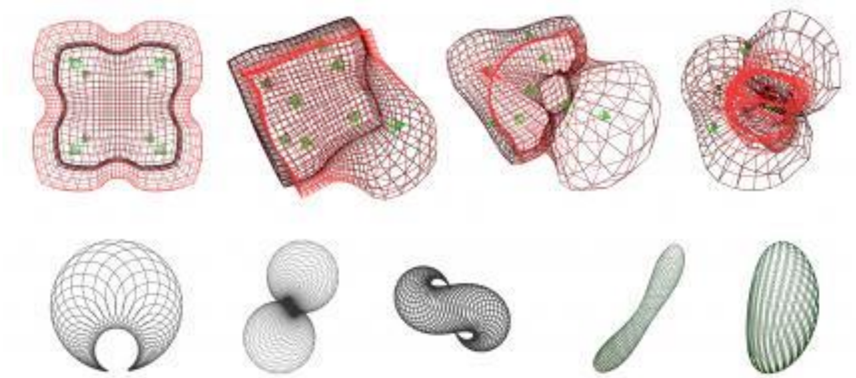
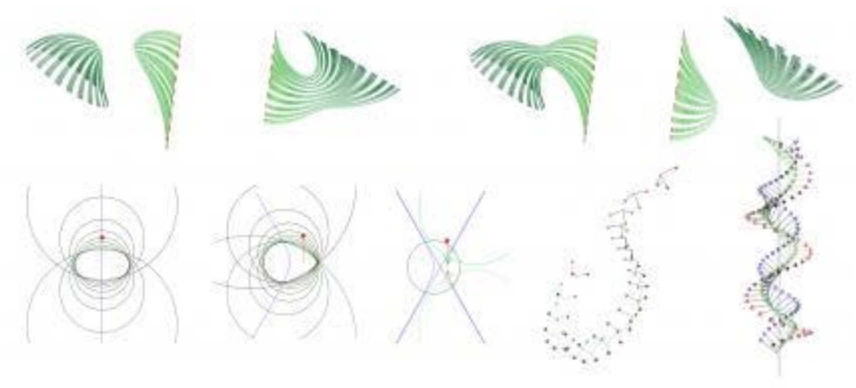
VERSOR: PROPOSAL FOR A SYSTEM OF ORGANIC CONSTRUCTIVISM

Pablo Colapinto

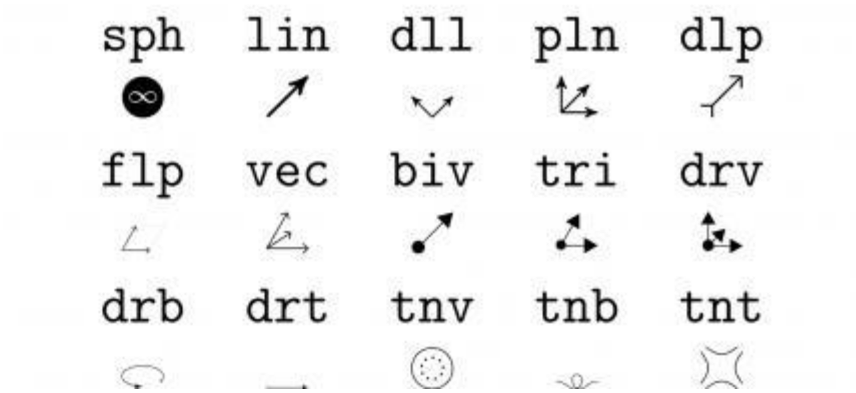
An introduction to the characteristics of conformal geometric algebra [CGA] is used to argue for its potential use in morphogenetic research and experimentation by digital artists. Figures included were generated using *Versor*, interactive software for generating organic forms and environments.



1. Above: The effect of a warp field on a lattice of points. Below: Some loxodromic transformations of a circle.



2. Above: Soap-bubble effect. A model of spontaneous surface generation through linear combinations of circles. Below Left: Some conformal transformations of a circle. Below Right: Motor generating twists.



3. Table of elements and operators in 5 dimensional conformal geometric algebra. Three-letter codes are used for shorthand and represent the origin, infinity, point, point pair, circle, sphere, line, dual line, plane, dual plane, flat point, vector, bivector, trivector, directions, tangents, rotors, translators, dilators, motors, and transversors.

In these pages, I introduce a little-known mathematical framework for spatial computing, and suggest its particular suitability for constructing developmental systems analogous to those studied in biological research. I do so here without the use of equations – for a more in depth look at the algebra the reader is referred to my master's thesis on the subject, [1] and the references found therein.

The accompanying images are selections of processes generated using computer software I develop called *Versor*. *Versor* is a free and open source C++ graphics-synthesis toolset for exploring new techniques in the manipulation and activation of virtual forms and dynamic environments. It implements conformal geometric algebra through an efficient and integrated multimedia platform, low-level enough for “serious” applications and high-level enough for “user friendly” functionality. Drawing from an excellent textbook by L. Dorst, D. Fontijne, and S. Mann, [2] *Versor* simplifies experimental exploration by putting some of the more exquisite features of CGA into the hands of digital practitioners, and can be used for both artistic and engineering-based design. Integrated with various dynamic solvers, a graphics user interface library (GLV) [3] and an audio synthesis library (Gamma), [4] *Versor,* can be compiled as a stand alone application or as an external library. It incorporates many compositional techniques for analysis and synthesis of dynamic forms and structures, some drawn from the CGA research landscape and some introduced for the first time, such as “Hyper Fluids”.

Versor aims to provide a basis for research requiring the visualization of complex multidimensional fields such quantum electrodynamic phase spaces, gauge theories, lorentz fields, spacetime curvature tensors, and morphogenetics. Potential artistic and engineering uses include live performances, immersive environments, multimodal interfaces, molecular modeling and crystallography, morphological studies, hyperbolic geometry, and dynamic physics simulations. Its future development is suggested by the examination that follows – namely, the construction of an operational network for simulated ontogenetic processes.

Background

Geometric algebra [GA] is a mathematical system of combinatoric spatial logic based on William Clifford’s hypercomplex algebras of the 19th century. With it, synthetic geometric concepts such as circles and lines can become variables in an equation. The algebra allows one to use ratios of these geometric entities to construct analytical operations such as rotations or twists. More advanced concepts quickly emerge from this holistic and scalable combination of synthetic and analytic geometries.

A long-awaited fulfillment of "common sense" spatial relationships, GA integrates various methods for modeling and engineering dynamic systems. Applications exist in computer vision and graphics, neural nets, DSP, robotics, optics, particle and relativistic physics, and metamaterials research. The now classic work by the physicist David Hestenes, *New Foundations for Classical Mechanics* (1986) demonstrates the compactness of the math, [5] while the most recent text by cyberneticist Eduardo Bayro-Corrochano, *Geometric Computing: For Wavelet Transforms, Robot Vision, Learning, Control and Action* (2010), demonstrates the expressivity of its powers for geometric reasoning. [6] Many other authors make a similar case: geometric algebras encapsulate many other mathematical systems (through *isomorphisms*) and solutions worked out in smaller dimensions can often be extrapolated to higher dimensions (through *outermorphisms*). It is an expressive logic of spatial relationship, which allows intuitive mathematical experimentation across a widening range of disciplines.

Visible use of geometric algebra in spatial computing has grown with the introduction of a model for generalized homogenous coordinates. Based on Riemannian projection onto a hypersphere, physicists developed a conformal mapping of 3-dimensional Euclidean space onto a 5-dimensional Minkowskian sum. Introduced into the geometric algebra community by Hongbo Li, Alan Rockwood, and David Hestenes in 2001, [7] this conformal model enables algebraic manipulation of direct geometric entities – lines, planes, circles, spheres – as well as their dual representations and even more subtle concepts – surface tangents, tangent spaces, hyperplanes, and space-time boosts. It provides mechanisms for describing closed form (i.e. exact) solutions within Euclidean, spherical, and hyperbolic geometries, and unprecedented control over the synthesis of parametric forms. Still, the use of conformal geometric algebra for exploration in graphic modelling remains a relatively esoteric exercise in the larger scientific and artistic community. As a result, many of its formal characteristics and exotic morphological powers have yet to be fully explored.

Axiom and Organism

At the core of the mathematical system is the almost promethean enfranchisement of geometric primitives. Points, lines, planes, circles, and spheres can now operate on each other to create other entities. Three basic operators are defined – geometric product, wedge (outer product), and contraction (inner product) – as are three modifiers – reversion, involution, and conjugation. Elements can be modified to produce their inverse, thus providing a means for division. For instance, any element can be divided by the space it is in (referred to as the tangent space) to produce a dual representation of itself, which can be used in marvelous ways. For instance, a dual plane can be wedged together with a dual sphere to produce a dual circle – defining the meet between the plane and the sphere, or in other words, revealing the circle the plane cuts through the sphere.

All this is done algebraically, which means we can make computers do it. This is no small feat, for it endows computers with a rigorous toolset for deductive reasoning about space. And beyond introducing a cornucopia of different geometric elements representable as variables in an equation, this algebraic model of space simplifies and generalizes calculations of all general rigid body movements within Euclidean and non-Euclidean spaces. Conformal geometric algebra opens the door to a rich set of transformations previously unexplored visually and typically closed to artistic experimentation. These tranformational operators are called 'versors.' They allow for reflections, rotations, translations, dilations, motor twists, and transversion boosts around, along and across round and flat elements.

This powerful fusion of deductive reasoning with analytical techniques is an irresistible invitation to revisit the most confounding and particular shape-making and shape-changing processes known to date: the evolution (phylogeny) and growth (ontogeny) of living organisms. Indeed, beyond its logical formalism, conformal geometric algebra exhibits certain characteristics, which mark it as a peculiarly organic.

Combinatorics and Hypercomplexity

Geometric algebras rely upon a combinatoric system to generate a hypercomplex graded algebra of multivectors. These sparse matrices of mixed dimensionality are ultimately what represent our transformational operators and geometric primitives. Hypercomplexity here refers to the fact that there multiple imaginary numbers involved in the system. That is the concept of multidimensional hypercomplex numbers should not be confused with the concept of complex or nonlinear behaviors which exhibit emergent properties, except in so far as they can be more easily be used to generate such behaviors algorithmically. Here, I should note an intriguing set of articles written by C. Muses in the late 1970’s which point to hypercomplex numbers as having a unique use in modeling biological systems. Published under the auspices of the mobile and mysterious “Research Centre for Mathematics and Morphology,” Muses’ articles – such as the 1979*Computing in the Bio-Sciences with Hypernumbers: A Survey* – are filled with parametric lobes and coils and argue for the specialized use of imaginary numbers in the study of biological form.

Point and Process

The mechanics of the conformal model begins with the definition of a null vector – a point in space which has has no magnitude. Using Hermann Grassman's algebra of extensions, it builds up larger dimensional objects such as point pairs, circles, and spheres. These round elements of the algebra serve as both operators and objects in a consistent and predictable way. One can construct a continuously differentiable perturbation operator from a pair of points that is itself perturbed by another operator. This greatly simplifies dynamic simulations by providing a linear approach to creating higher order phenomena. Since there is no real difference, in the computer's memory, between a point in space and a process in space, the representation of space itself gains a certain parameterizable agency. For instance, the construction of self-organizing tangent spaces is possible, creating a whirlwind of subspaces within it: a spatial configuration of configuration spaces.

Twist and Boost

The plethora of round elements created by combinatorics would by itself provide a rather glib organicity were it, not for the particular elegance of the operations allowed. Given a line in space, we can twist any multivector (geometric element) around it. Given a point in space, we can accelerate any multivector into it or away from it. These transformations describe rich movements in space that can be easily compounded. The versors form a closed *automorphic* group, such that multiplying them together will return another member of the group. This powerful structure allows for transformations to be concatenated through multiplication as is done in matrix algebra. The complete set of Euclidean and conformal transformations are possible with the versor construction: inversions, translations, rotations, screw motions, dilations, and transversions. By constructing an operational network of transformational effects, bilbous and twisty forms are easily generated. Spaces can be cinched, pinched, blown out, twirled, twisted, or torn into and out of real and imaginary radii.

Morphisms and Hybridity

Tensor, vector, and matrix algebras are all embedded in geometric algebra through characteristics shared with group theory and lie algebras. A variety of mathematical species such as complex numbers, Plücker coordinates, Dirac and Pauli Matrices, and the symmetries of various particles as described by lie groups and their algebras have been shown to be isomophic to, and easily represented by, various metrics of geometric algebra. These isomorphisms are critical to cross-fertilization across different fields of science, bridging gaps between physicists and biologists, for instance. The algebra also has outermorphic properties: discoveries made with simpler elements in lower dimensions can often be generalized to higher dimensions. The logic is designed to be extrapolated meaningfully, and the outermorphic properties of the algebra allow for this. This helps in building intuition and experimenting with algorithms across dimensions.

Orientation and Polarity

A critical component of the algebra is its asymmetric anticommutivity, for this fuels its orientability. Circles are not just circles, but they have an inherent direction (clockwise or anticlockwise). Spheres likewise have an inherent spin, as do lines a direction. The orientability of the algebra is useful for describing chiralities or "handedness". This sensitivity to the polarity of forms is also found in the molecules of living organisms.

What these analogous characteristics point to is the suggestion that if any mathematical system is appropriate for modeling characteristics found in living organisms, it is some flavor of geometric algebra. What, then, is proposed here is the use of these grammar-of-forms in the design of an ontogenic processor: a simulated developmental system. Ontogeny is the trajectory of an individual organism's morphogenetic developments, from seed to maturation and death. Phylogeny is the evolution of ontogeny over generations. Embedding the GA grammar into a functional framework of emergent causal structures could be a first step towards a phylogenic-ontogenic computer.

What is currently missing is a sort of statistical memory. In a computer we are forced to use memory in ways that it is never used in biology, so it is important to set a few internal boundaries for this task. It is not the proposed goal to envision a geometry that can correctly model any particular specimen. This limitation is set to avoid answering the question of how real life forms grow, the mechanics of which are best described by biologists. Similarly, it is not the intention to reflect a model of life back onto actual life in a particular way – that is, to make a specific statement about living things that would be better phrased by philosophers.

Rather, the primary goals are artistic: an aesthetics of movement, a self-organizing differentiation of forms and tangent spaces, a furnishing of lifelike qualities into a geometric system, the further enfranchisement of uncertain spatial concepts, the development of new relations. Developmental analysis remains one of the most mysterious biological processes, and I doubt we will ever fully simulate the growth of a seed in computer memory. We might, however, be able to do justice to a dream of adaptation and growth: to generate a network of spatial relations complex enough to grow *something*. Then, precisely where and how the computer's analogies fail to illuminate actual biological processes could prove fruitful in both biological and philosophical trains of thought.

References and Notes:

1. *Pablo Colapinto, "Versor: Spatial Computing with Conformal Geometric Algebra" (Master's Thesis, University of California at Santa Barbara, 2011). Available at http://www.wolftype.com/versor (accessed September 1, 2011).*
2. *Leo Dorst, Daniel Fontijne and Stephen Mann, Geometric Algebra for Computer Science (Burlington, MA: Morgan Kaufmann Publishers, 2007).*
3. *Lance Putnam, Graham Wakefield and Eric Newman, "GLV: A C++ GUI Toolkit," GLV Official Website, http://www.mat.ucsb.edu/glv/ (accessed September 1, 2011).*
4. *Lance Putnam, "Gamma – Generic Synthesis Library," Gamma Official Website, http://www.mat.ucsb.edu/gamma/ (accessed September 1, 2011).*
5. *David Hestenes, New Foundations for Classical Mechanics (Boston: D. Reide., 1998).*
6. *Eduardo Bayro-Corrochano, Geometric Computing: For Wavelet Transforms, Robot Vision, Learning, Control and Action (London: Springer-Verlag, 2010).*
7. *Hongbo Li, David Hestenes and A. Rockwood, "Generalized Homogeneous Coordinates for Computational Geometry," in Geometric Computing with Clifford Algebra, ed. Gerald Sommer, (Berlin: Springer-Verlag, 2001) , 25-28.*